J Stat Phys (2010) 141: 952–956
DOI 10.1007/s10955-010-0092-8

Game of Life on the Equal Degree Random Lattice

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Received: 11 May 2010 / Accepted: 1 November 2010 / Published online: 11 November 2010

Abstract An effective matrix method is performed to build the equal degree random (EDR) lattice, and then a cellular automaton game of life on the EDR lattice is studied by Monte Carlo (MC) simulation. The standard mean field approximation (MFA) is applied, and then the density of live cells is given $\rho = 0.37017$ by MFA, which is consistent with the result $\rho = 0.37 \pm 0.003$ by MC simulation.

Keywords Game of life · Random lattice · Equal degree

1 Introduction

The cellular automaton has been extensively studied in many years due to its relevant applications in many social, biological, and physical processes [1–5]. Game of Life (GL) proposed by Conway is probably most famous cellular automaton [6]. GL has been applied to mimic aspects of complexity in nature [7–15]. The original GL Life is a 2-state cellular automaton defined on a square 2-Dimensional (2D) lattice, where each cell is alive or dead. The evolution is determined by the number of living cells among its Moore neighborhood (nearest and next-to-nearest neighbors): (1) Each live cell will remain alive in the next time step if it has two or three live neighbors, otherwise it will die; (2) Each dead cell will be alive only if there are exactly three live neighbors. Starting from random initial conditions, "Life" will evolve through complex patterns eventually settling down in a stationary state [6–8, 16–18]. In spite of its simple algorithm, the GL simulates the dynamic evolution of a society of living individuals, including processes such as growth, death, survival, self-propagation, and competition.

The GL on the 2D lattice had been analyzed by mean field approximation (MFA) [17, 18], where the correlation in the evolution of life is disregarded. Density of live cells is

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defined as the ratio of living cells to the total number of cells in the lattice. Then, the stable point of density of live cells is obtained $\rho = 0.37017$. However, by using Monte Carlo (MC) simulations, $\rho = 0.029 \pm 0.001$ is observed. The big difference comes from the approximation, that the cells are uncorrelated in the MFA [17]. However, the 2D square lattice is correlated in the MC simulations. In recent years, many social, biological, engineering and communication systems may be modeled as complex lattice, whose nodes represent individuals or organizations and links mimic the interactions among them [19–21]. One example is the small-world lattice proposed by Walts and Strogatz (WS) [22]. Huang et al. developed the GL on a 2D small-world network to study effect of network topology on the evolved dynamics of GL [16]. When the network randomness equals one, the 2D small-world network is completely uncorrelated, but the density of live cells $\rho = 0.345$ obtained by MC simulations [16]. The result is still different from one analyzed by MFA.

In this paper, we create a kind of equal degree random (EDR) lattice and study the game of Life on this lattice. It is found that simulation results about game of Life on this lattice can be consistent with the ones by MFA.

It is useful to give a matrix representation of a graph. The graph $G = (L, \varphi)$ can be completely described by giving the adjacency (or connectivity) matrix A, where L is the set of nodes, and φ is the set of links. The matrix A is a $L \times L$ square matrix whose entry a_{ij} $(i, j = 1, \dots, L)$ equals 1 for the link, otherwise zero. Here, we give a efficient method to build a EDR lattice with degree K as follow: N is taken even number. $b_1, \dots, b_K$ are different numbers randomly chosen from $[1, N/2]$, and then all i and j which satisfy that $i + j = N + 1 + 2 \times (b_m - 1)$, or $i + j = 1 + 2 \times (b_m - 1)$, m is taken from 1 to k, then the entry $a_{ij} = 1$; else $a_{ij} = 0$. We calculate the clustering coefficient of the created lattice; it is found that the clustering coefficient is zero. So we obtain a random adjacency matrix A, which is a EDR lattice with degree K without self link and multiple link.

The GL in the EDR lattice is described as follows: each cell in the lattice may be in two states, representing alive or dead. In the model, we define that two cells are neighboring if there exists a link between them. The state of each cell depends on its neighbors. The rule of evolution is: a dead individual will be alive, if it has exactly three living neighbors; a living individual will remain alive, if it has two or three living neighbors, otherwise it will die.

We start at time $t = 0$ with a random distribution of living cells with density ρ_0. As a standard cellular automaton procedure, at each time step all cells are updated simultaneously according to the rule described above, until the GL reaches a stationary state. The MC simulations are performed with $k = 8$ for the extensive system sizes ranging from $L = 10^3$ to $L = 10^6$. For every system with size L, the calculated results are averaged over both n $(n \times L = 10^7)$ different realizations of the EDR lattice and 10 independent ρ_0 runs for each lattice realization.

Figure 1 shows the density of live cells $\rho(t)$ as a function of time t in the lattice for $\rho_0 = 0.2$, 0.35, and 0.5 with the system with size $L = 10^3$. As we can see, the density of live cells $\rho(t)$ will approach a stable value for various initial density of live cells ρ_0. In Fig. 2, we plot the stable point ρ as a function of the initial density of live cells ρ_0. As shown in Fig. 2, the stable point of density of live cells is $\rho = 0.37 \pm 0.003$. Figure 3 shows the stable point of density of live cells ρ as function of system size L. As shown in Fig. 3, the stable point of density of live cells ρ is independent of the system size L.

The standard MFA [17, 18] is used to analyze the GL on the EDR lattice. S_i is the state of a cell at time t in the location i. According to the evolution rule, the equation of evolution can be expressed as

$$S_i(t + 1) = \delta(H_i^t, 3) + S_i(t)\delta(H_i^t, 2), \tag{1}$$

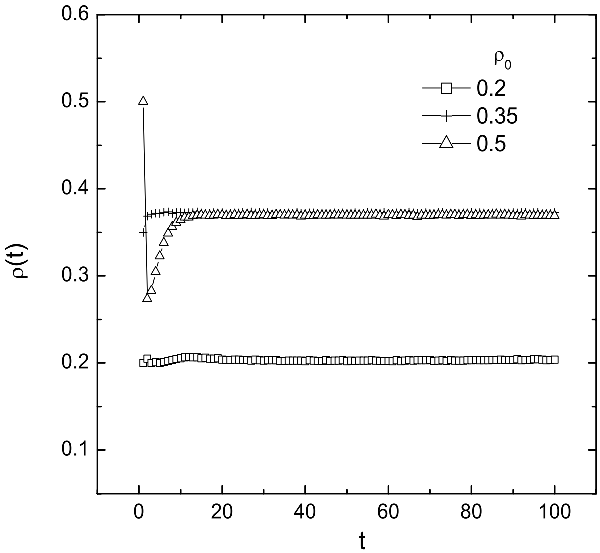

Fig. 1 The density of live cells $\rho(t)$ as a function of time t at the lattice for $\rho_0 = 0.2$, 0.35, and 0.5 with the system with size $L = 10^4$

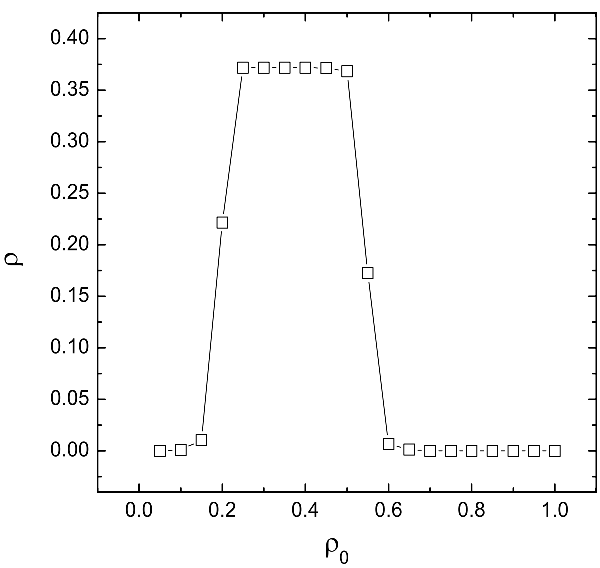

Fig. 2 The stable point ρ as a function of the initial density of live sites ρ_0

where H_i^t is the number of living cells among its neighborhood at time t. In the EDR lattice, two nodes are neighborhood if there is a link between them. δ is the Kronecker symbol, $\delta(m,n) = 1$ if $m = n$, 0 otherwise.

The evolution of density of live cells $\rho(t) = \frac{\sum_i S_i(t)}{L}$ is found by averaging (1). The average of the first term $\delta(H_i^t, 3)$ is the probability of the number of living cells among its neighborhood is exactly three live neighbors. Given that the overall density of live cells is $\rho(t)$, the probability that for any eight neighbors there are exactly three live ones among them (and in the absence of correlations) is $C_8^3(\rho(t)^3(1-\rho(t))^5$. Reasoning similarly for the last term $\delta(H_i^t, 2)$ in (1), the probability that for any eight neighbors there are exactly three live ones among them (and in the absence of correlations) is $C_8^2(\rho(t)^2(1-\rho(t))^6$.

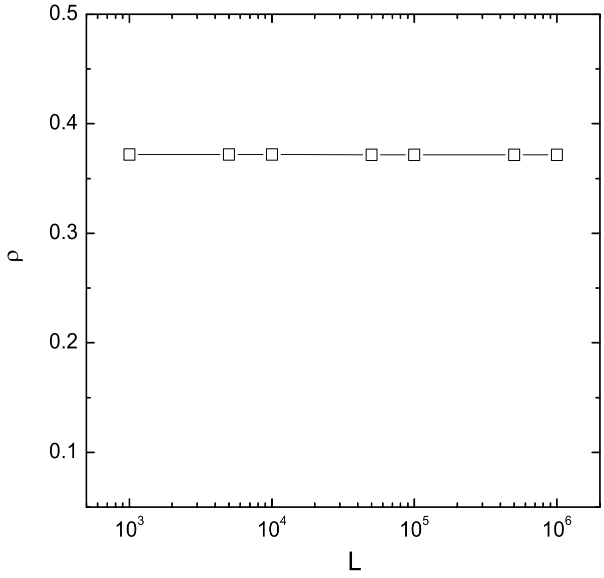

Fig. 3 The stable point of density of live cells ρ as function of system size L

These lead to the following evolution equation for the EDR lattice with $k = 8$

$$\rho(t+1) = C_8^3(\rho(t)^3(1-\rho(t))^5 + \rho(t)C_8^2(\rho(t)^2(1-\rho(t))^6, \tag{2}$$

where t is the time step. And then,

$$\rho(t+1) = 28(\rho(t)^3(1-\rho(t))^5(3-\rho(t)). \tag{3}$$

If the system has a long-term steady state, then $\rho(t+1)$ will equal $\rho(t)$. Setting $\rho(t+1) = \rho(t)$ in (3) yields an equation for $\rho(t)$ whose real solutions are $\rho = 0$ and $\rho = 0.37017$. Without correlations, the system moves to the density of live cells given by the fixed points of (3). Then the stable point of ρ is 0.37017, which is consistent with the results obtained from MC simulation. The results show that the MFA is a good approximation but not exact. The reason is that because the adjacency matrix is symmetric, there is no way to avoid correlations. Already after two time steps, the correlations should have an effect, as a site influences its neighbors, so that the neighbors get correlated and these correlated neighbors influence back the site. Since the number of neighbors is large and so the effect of correlations is reduced, the simulation results agree with MFA.

In conclusion, an effective matrix method to build the EDR lattice is proposed. Game of life on the lattice is studied by MC simulation. The standard MFA is applied in the model, and the density of live cells is given $\rho = 0.37017$, which is consistent with the result $\rho = 0.37 \pm 0.003$ obtained from MC simulation.

Acknowledgements The author would like to thank the anonymous referees for valuable hints and constructive suggestions to improve the presentation of the results. This work was supported by the South China Normal University with Grant No. S53003 and Foundation for Distinguished Young Talents in Higher Education of Guangdong, China.

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